# First day. Juniors

*14 March*

**1.** Basil needs to solve an exercise on summing two fractions $\frac{a}{b}$ and $\frac{c}{d}$, where $a$, $b$, $c$, $d$ are some non-zero real numbers. But instead of summing he performed multiplication (correctly). It appears that Basil's answer coincides with the correct answer to given exercise. Find the value of $\frac{b}{a} + \frac{d}{c}$.

*folklore, edited by P. Kozhevnikov*

**2.** On Mars a basketball team consists of 6 players. The coach of the team Mars can select any line-up of 6 players among 100 candidates. The coach considers some line-ups as *appropriate* while the other line-ups are not (there exists at least one appropriate line-up). A set of 5 candidates is called *perspective* if one more candidate could be added to it to obtain an appropriate line-up. A candidate is called *universal* if he completes each perspective set of 5 candidates (not containing him) upto an appropriate line-up. The coach has selected a line-up of 6 universal candidates. Determine if it follows that this line-up is appropriate.

*V. Bragin*

**3.** Find the least positive integer $n$ satisfying the following statement: for eash pair of positive integers $a$ and $b$ such that 36 divides $a+b$ and $n$ divides $ab$ it follows that 36 divides both $a$ and $b$.

*PSC, based on the problem from Polish olympiad*

**4.** In an acute traingle $ABC$ with $AB < BC$ let $BH_b$ be its altitude, and let $O$ be the circumcenter. A line through $H_b$ parallel to $CO$ meets $BO$ at $X$. Prove that $X$ and the midpoints of $AB$ and $AC$ are collinear.

*P. Kozhevnikov*

# Second day. Juniors

*15 March*

**5.** In a football tournament 20 teams participated, each pair of teams played exactly one game. For the win the team is awarded 3 points, for the draw — 1 point, for the lose no points awarded. The total number of points of all teams in the tournament is 554. Prove that there exist 7 teams each having at least one draw.

*D. Belov, based on folklore problems*

**6.** A triangle is cut by 3 cevians from its 3 vertices into 7 pieces: 4 triangles and 3 quadrilaterals. Determine if it is possible that all 3 quadrilaterals are inscribed.

*A. Trufanov*

**7.** 10 distinct numbers are given. Professor Odd had calculated all possible products of 1, 3, 5, 7, 9 numbers among given numbers, and wrote down the sum of all these products. Similarly, Professor Even had calculated all possible products of 2, 4, 6, 8, 10 numbers among given numbers, and wrote down the sum of all these products. It appears that Odd's sum is greater than Even's sum by 1. Prove that one of 10 given numbers is equal to 1.

*P. Kozhevnikov, based on folklore problems*

**8.** 100 points are marked in the plane so that no three of marked points are collinear. One of marked points is red, and the others are blue. A triangle with vertices at blue points is called *good* if the red point lies inside it. Determine if it is possible that the number of good triangles is not less than the half of the total number of traingles with vertices at blue points.

*P. Kozhevnikov*

# First day. Seniors

*14 March*

**1.** Two points $A$ and $B$ lie on two branches of hyperbola given by equation $y = \frac{1}{x}$. Let $A_x$ and $A_y$ be projections of $A$ onto coordinate axis, similarly, let $B_x$ and $B_y$ be projections of $B$ onto coordinate axis. Prove that triangles $AB_xB_y$ and $BA_xA_y$ have equal areas.

*N. Avilov*

**2.** On Mars a basketball team consists of 6 players. The coach of the team Mars can select any line-up of 6 players among 100 candidates. The coach considers some line-ups as *appropriate* while the other line-ups are not (there exists at least one appropriate line-up). A set of 5 candidates is called *perspective* if one more candidate could be added to it to obtain an appropriate line-up. A candidate is called *universal* if he completes each perspective set of 5 candidates (not containing him) upto an appropriate line-up. The coach has selected a line-up of 6 universal candidates. Determine if it follows that this line-up is appropriate.

*V. Bragin*

**3.** In an acute traingle $ABC$ with $AB < BC$ let $BH_b$ be its altitude, and let $O$ be the circumcenter. A line through $H_b$ parallel to $CO$ meets $BO$ at $X$. Prove that $X$ and the midpoints of $AB$ and $AC$ are collinear.

*P. Kozhevnikov*

**4.** Determine if there exist 101 positive integers (not necessarily distinct) such that their product is equal to the sum of all their pairwise LCM.

*S. Tokarev*

# Second day. Seniors

*15 March*

**5.** In a football tournament 20 teams participated, each pair of teams played exactly one game. For the win the team is awarded 3 points, for the draw — 1 point, for the lose no points awarded. The total number of points of all teams in the tournament is 554. Prove that there exist 7 teams each having at least one draw.

*D. Belov, based on folklore problems*

**6.** Given real numbers $a$, $b$, $c$ satisfy inequality $\left|\frac{a^2+b^2-c^2}{ab}\right| < 2$. Prove that they also satisfy equalities $\left|\frac{b^2+c^2-a^2}{bc}\right| < 2$ and $\left|\frac{c^2+a^2-b^2}{ca}\right| < 2$.

*L. Emelyanov*

**7.** 8 ants are placed on the edges of the unit cube. Prove that there exists a pair of ants at a distance not exceeding 1.

*Folklore, edited by S. Tokarev*

**8.** Given a table in a form of the regular 1000-gon with sidelength 1. A Beetle initially is in one of its vertices. All 1000 vertices are numbered in some order by numbers $1, 2, \ldots, 1000$ so that initially the Beetle is in the vertex 1. The Beetle can move only along the edges of 1000-gon and only clockwise. He starts to move from vertex 1 and he is moving without stops until he reaches vertex 2 where he has a stop. Then he continues his journey clockwise from vertex 2 until he reaches the vertex 3 where he has a stop, and so on. The Beetle finishes his journey at vertex 1000. Find the number of ways to enumerate all vertices so that the total length of the Beetle's journey is equal to 2017.

*P. Kozhevnikov*