VII Caucasus Mathematic Olympiad
Maykop, March 11–16, 2022 year

Caucasus Mathematical Olympiad | Кавказская математическая олимпиада

Juniors. Day 1
March 12

1. Positive integers a, b, c are given. It is known that $\frac{c}{b} = \frac{b}{a}$, and the number $b^2 - a - c + 1$ is a prime. Prove that a and c are doubled squares of positive integers.

2. In parallelogram $ABCD$, points E and F on segments AD and CD are such that $\angle BCE = \angle BAF$. Points K and L on segments AD and CD are such that $AK = ED$ and $CL = FD$. Prove that $\angle BKD = \angle BLD$.

3. Pete wrote down 21 pairwise distinct positive integers, each not greater than $1,000,000$. For every pair (a, b) of numbers written down by Pete, Nick wrote the number

$$F(a; b) = a + b - \gcd(a; b)$$

on his piece of paper. Prove that one of Nick's numbers differs from all of Pete's numbers.

4. Do there exist 2021 points with integer coordinates on the plane such that the pairwise distances between them are pairwise distinct consecutive integers?

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Seniors. Day 1
March 12

1. Given a rectangular table with 2 rows and 100 columns. Dima fills the cells of the first row with numbers 1, 2 or 3. Prove that Alex can fill the cells of the second row with numbers 1, 2, 3 in such a way that the numbers in each column are different and the sum of the numbers in the second row equals 200.

2. Prove that infinitely many positive integer numbers can be represented as $(a-1)/b+(b-1)/c+(c-1)/a$, where a, b and c are pairwise distinct natural numbers greater than 1.

3. Do there exist 100 points on the plane such that the pairwise distances between them are pairwise distinct consecutive integer numbers larger than 2022?

4. Circle ω is tangent to the sides of an acute angle with vertex A at points B and C. Let D be an arbitrary point on the major arc BC of the circle ω. Points E and F are chosen inside the angle DAC so that the quadrilaterals $ABDF$ and $ACED$ are inscribed and the points A, E, F lie on the same straight line. Prove that the lines BE and CF intersect at ω.