VII Caucasus Mathematic Olympiad
Maykop, March 11–16, 2022 year

Caucasus Mathematical Olympiad | Кавказская математическая олимпиада

Juniors. Day 2

March 13

5. Let S be the set of all 5^6 positive integers, whose decimal representation consists of exactly 6 odd digits. Find the number of solutions (x, y, z) of the equation $x + y = 10z$, where $x \in S$, $y \in S$, $z \in S$.

6. 16 NHL teams in the first playoff round divided in pairs and to play series until 4 wins (thus the series could finish with score 4-0, 4-1, 4-2, or 4-3). After that 8 winners of the series play the second playoff round divided into 4 pairs to play series until 4 wins, and so on. After all the final round is over, it happens that k teams have non-negative balance of wins (for example, the team that won in the first round with a score of 4-2 and lost in the second with a score of 4-3 fits the condition: it has $4 + 3 = 7$ wins and $2 + 4 = 6$ losses). Find the least possible k.

7. Point P is chosen on the leg CB of right triangle ABC ($\angle ACB = 90°$). The line AP intersects the circumcircle of ABC at point Q. Let L be the midpoint of PB. Prove that QL is tangent to a fixed circle independent of the choice of point P.

8. Paul can write polynomial $(x + 1)^n$, expand and simplify it, and after that change every coefficient by its reciprocal. For example if $n = 3$ Paul gets $(x + 1)^3 = x^3 + 3x^2 + 3x + 1$ and then $x^3 + \frac{1}{3}x^2 + \frac{1}{3}x + 1$. Prove that Paul can choose n for which the sum of Paul's polynomial coefficients is less than 2.022.

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Seniors. Day 2

March 13

5. 16 NHL teams in the first playoff round divided in pairs and to play series until 4 wins (thus the series could finish with score 4-0, 4-1, 4-2, or 4-3). After that 8 winners of the series play the second playoff round divided into 4 pairs to play series until 4 wins, and so on. After all the final round is over, it happens that k teams have non-negative balance of wins (for example, the team that won in the first round with a score of 4-2 and lost in the second with a score of 4-3 fits the condition: it has $4+3=7$ wins and $2+4=6$ losses). Find the least possible k.

6. Let ABC be an acute triangle. Let P be a point on the circle (ABC), and Q be a point on the segment AC such that $AP \perp BC$ and $BQ \perp AC$. Let O be the circumcenter of triangle APQ. Find the angle OBC.

7. Paul can write polynomial $(x+1)^n$, expand and simplify it, and after that change every coefficient by its reciprocal. For example if $n=3$ Paul gets $(x+1)^3 = x^3+3x^2+3x+1$ and then $x^3+\frac{1}{3}x^2+\frac{1}{3}x+1$. Prove that Paul can choose n for which the sum of Paul's polynomial coefficients is less than 2.022.

8. There are $n > 2022$ cities in the country. Some pairs of cities are connected with straight two-ways airlines. Call the set of the cities *unlucky*, if it is impossible to color the airlines between them in two colors without monochromatic triangle (i.e. three cities A, B, C with the airlines AB, AC and BC of the same color).

The set containing all the cities is unlucky. Is there always an unlucky set containing exactly 2022 cities?